Math 308N **Midterm 2** **Winter 2018**

Your Name

Your Signature

Student ID #

Honor Statement

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: ______________________________

- Turn off all cell phones, pagers, radios, mp3 players, and other similar devices.
- This exam is closed book. You may use one $8.5'' \times 11''$ sheet of handwritten notes (both sides OK). Do not share notes. No photocopied materials are allowed.
- Only the TI 30X IIS calculators is allowed.
- In order to receive credit, you must **show all of your work**. If you do not indicate the way in which you solved a problem, you may get little or no credit for it, even if your answer is correct.
- If you need more room, use the backs of the pages and indicate that you have done so.
- Raise your hand if you have a question.
- This exam has 7 pages, plus this cover sheet. Please make sure that your exam is complete.

Question	Points	Score
1	18	
2	10	
3	14	
4	10	
5	8	
Total	60	

1. (18 points) Indicate whether the given statement is true or false (2 pts) and give justification as to why it is true or false(2 pts).

a) [4 pts] If S is a subspace of $\mathbb{R}^8$ and $\{\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_7\}$ is a basis for S, then for any $\vec{v} \notin S$, $\text{span}\{\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_7, \vec{v}\} = \mathbb{R}^8$.

b) [4 pts] Let B_1, B_2, and B_3 be bases for $\mathbb{R}^n$. If C_1 is the change of basis matrix going from B_1 to B_2, and C_2 is the change of basis matrix going from B_2 to B_3, then $C_2^{-1}C_1^{-1}$ is the change of basis matrix going from B_3 to B_1.

c) [4 pts] If W is a subspace of $\mathbb{R}^9$, $dim(W) = 3$, and $T : \mathbb{R}^9 \to \mathbb{R}^6$ is a linear transformation such that Ker(T)=W, then T must be onto.

Give an example of each of the following. If it is not possible write "NOT POSSIBLE", and **give justification as to why**.

d) [2 pt] A 2×2 matrix $A \neq I_2$ such that $A^{2018} = I_2$, but $A^k \neq I_2$ for all $k < 2018$.

e) [2 pt] A linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ such that range(T)=ker(T).

f) [2 pts] A basis B for $\mathbb{R}^3$ such that every vector lies in the set $\left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a + b + 2c = 0 \right\}$

2. (10 points) Consider the matrix A, and its reduced echelon form below

$$A = \begin{bmatrix} 2 & -6 & 14 & 4 & 18 \\ -1 & 6 & -19 & 4 & -6 \\ -2 & 7 & -18 & 1 & -11 \\ 3 & -8 & 17 & 3 & 18 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -5 & 0 & -4 \\ 0 & 1 & -4 & 0 & -3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

a) [4 pts] Find a basis for $\text{Col}(A)$, the column space of A.

b) [4 pts] Find a basis for $\text{Null}(A)$, the Null space of A.

c) [2 pts] Find a basis for $\text{row}(A^T)$, the row space of A^T.

3. (14 points) a) [5 pts] Consider the set $S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : x^2 + y^2 + z^2 \leq 1 \right\}$. Determine if S is a subspace of $\mathbb{R}^3$. If it is, show that it is a subspace. If it is not, give justification as to why.

b) [5 pts] Let A be an $n \times n$ matrix and consider the set $S = \left\{ x \in \mathbb{R}^n : A\vec{x} = \vec{x} \right\}$. That is, for some fixed matrix A, S consists of all vectors in $\mathbb{R}^n$ that are fixed by A. Show that S is a subspace of $\mathbb{R}^n$. (Note: You may use the definition or any theorems from class.)

(Continued on next page $\Longrightarrow$)

c)[4pts] Given the set S from part b), find a basis of S for the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$

4. (10 points) a) [5 pts] Define linear transformations $T_A : \mathbb{R}^3 \to \mathbb{R}^3$, $S_B : \mathbb{R}^3 \to \mathbb{R}^3$, and $R = (T \circ S) : \mathbb{R}^3 \to \mathbb{R}^3$ with $T_A(\vec{x}) = A\vec{x}$ and $S_B(\vec{x}) = B\vec{x}$ for $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$.
   Show that T and S are invertible. (Note: Even though matrix B is the same as in the previous problem, they are unrelated.)

   b) [5 pts] Determine a matrix C such that $R^{-1}(\vec{x}) = (T \circ S)^{-1}(\vec{x}) = C\vec{x}$.

5. (8 points) Consider the following 5×5 matrices:

$$M = \begin{bmatrix} 2 & 5 & \sqrt[5]{3} & 2 & \sqrt{2} \\ -3 & 8 & 3 & -6 & 1 \\ \pi & 52 & e & 3 & 5 \\ \sqrt{3} & 2 & 9 & 4 & 7\sqrt{13} \\ 5 & \pi^4 & -1 & 3 & 2 \end{bmatrix} \qquad D = \begin{bmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & -2 \end{bmatrix}$$

An absolutely horrendous computation shows that M is invertible (You don't need to show this) so you may assume that M^{-1} exists. Define a new matrix $A = MDM^{-1}$. Is A invertible? If so, give a formula for A^{-1} as a product of matrices (You do **not** need to find an explicit formula for M^{-1}). Be sure to carefully explain your reasoning.